AP STATS NOTES [12.1] #1

NAME PER

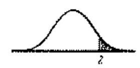
## z Test for a Population Mean

To test the hypothesis $H_0$: $\mu = \mu_0$ based on an SRS of size $n$ from a population with unknown mean $\mu$ and known standard deviation $\sigma$, compute the one-sample $z$ statistic

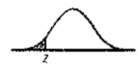
$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

In terms of a variable Z having the standard Normal distribution, the $P$-value for a test of $H_0$ against

$H_a$: $\mu > \mu_0$ is $P(Z \geq z)$




$H_a$: $\mu < \mu_0$ is $P(Z \leq z)$




$H_a$: $\mu \neq \mu_0$ is $P(Z \geq |z|)$


These $P$-values are exact if the population distribution is Normal and are approximately correct for large $n$ in other cases.

1. The one-sample z statistic for a one-sided test has the value z = - 1.78
Find the P-value for the test.
Is this significant at the 10% level?
Is this significant at the 5% level?

2. The one-sample z statistic for a one-sided test has the value z = 2.73
Find the P-value for the test.
Is this significant at the 5% level?
Is this significant at the 1% level?

3. The one-sample z statistic for a two-sided test has the value z = 1.40
Find the P-value for the test.
Is this significant at the 10% level?
Is this significant at the 5% level?

## The One-Sample t Statistic and the t Distributions

Draw an SRS of size $n$ from a population that has the Normal distribution with mean $\mu$ and standard deviation $\sigma$. The **one-sample $t$ statistic**

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

has the $t$ distribution with $n - 1$ degrees of freedom.

**Using the "t table," I** What critical value $t^*$ from Table C satisfies each of the following conditions?

(a) The $t$ distribution with 9 degrees of freedom has probability .02 to the right of $t^*$.

(b) The $t$ distribution with 16 degrees of freedom has probability .95 to the left of $t^*$.

**Using the "t table," II** What critical value $t^*$ from Table C satisfies each of the following conditions?

(a) The one-sample $t$ statistic from a sample of 25 observations has probability .005 to the right of $t^*$.

(b) The one-sample $t$ statistic from an SRS of 14 observations has probability .8 to the left of $t^*$.

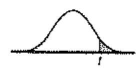
### *The One-Sample t Test*

Draw an SRS of size $n$ from a population having unknown mean $\mu$. To test the hypothesis $H_0$: $\mu = \mu_0$ based on an SRS of size $n$, compute the **one-sample $t$ statistic**

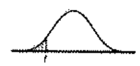
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

In terms of a random variable $T$ having the $t(n - 1)$ distribution, the $P$-value for a test of $H_0$ against

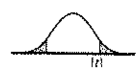
$H_a$: $\mu > \mu_0$ is $P(T \geq t)$



$H_a$: $\mu < \mu_0$ is $P(T \leq t)$



$H_a$: $\mu \neq \mu_0$ is $2P(T \geq |t|)$



These $P$-values are exact if the population distribution is Normal and are approximately correct for large $n$ in other cases.

**One-sided $t$ test** The one-sample $t$ statistic for testing

$$H_0: \mu = 0$$
$$H_a: \mu > 0$$

from a sample of $n$ = 20 observations has the value $t$ = 2.46

(a) What are the degrees of freedom for this statistic?

(b) Give the two critical values $t^*$ from Table C that bracket $t$ What are the upper tail probabilities $p$ for these two entries?

(c) Between what two values does the $P$-value of the test fall?

(d) Is the value $t$ = 2.46 significant at the 5% level? Is it significant at the 1% level?

**Two-sided $t$ test** The one-sample $t$ statistic from a sample of $n$ = 15 observations for the two-sided test of

$$H_0: \mu = 64$$
$$H_a: \mu \neq 64$$

has the value $t$ = 2.35

(a) What are the degrees of freedom for $t$?

(b) Locate the two critical values $t^*$ from Table C that bracket $t$. What are the upper tail probabilities $p$ for these two values?

(c) Between what two values does the $P$-value of the test fall? (Note that $H_a$ is two-sided.)

(d) Is the value $t$ = 2.35 statistically significant at the 10% level? At the 5% level?

**Using Table C** The one-sample $t$ statistic for a test of

$$H_0: \mu = 10$$
$$H_a: \mu < 10$$

based on $n$ = 30 observations has the value $t$ = −2.37

(a) What are the degrees of freedom for this statistic?

(b) Between what two probabilities from Table C does the $P$-value of the test fall?

**Significance** You are testing $H_0$: $\mu = 0$ against $H_a$: $\mu \neq 0$ based on an SRS of observations from a Normal population. What values of the $t$ statistic are statistically significant at the $\alpha$ = .025 level?

**Sample size and significance** For a sample of size 10 a test of a null hypothesis versus a two-sided alternative gives $t$ = 1.786

(a) Is the test result significant at the 5% level? Draw a sketch of the appropriate $t$ distribution and illustrate your calculation with this sketch.

(b) Now assume that the same test statistic was obtained for a sample size of $n$ = 20 Assess the statistical significance of the result and illustrate the calculation with a sketch. How did the statistical significance change with the sample size? Explain your answer.

AP STATS NOTES [12.1] #2 NAME PER

**Example 12.2** *Sweet cola*

Performing a one-sample *t* test

Diet colas use artificial sweeteners to avoid sugar. These sweeteners gradually lose their sweetness over time. Manufacturers therefore test new colas for loss of sweetness before marketing them. Trained tasters sip the cola along with drinks of standard sweetness and score the cola on a "sweetness scale" of 1 to 10. The cola is then stored for a month at high temperature to imitate the effect of four months' storage at room temperature. Each taster scores the cola again after storage. Our data are the differences (score before storage minus score after storage) in the tasters' scores. The bigger these differences, the bigger the loss of sweetness. Here are the sweetness losses for a new cola, as measured by 10 trained tasters:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2.0 | 0.4 | 0.7 | 2.0 | −0.4 | 2.2 | −1.3 | 1.2 | 1.1 | 2.3 |

Most are positive. That is, most tasters found a loss of sweetness. But the losses are small, and two tasters (the negative scores) thought the cola gained sweetness. *Are these data good evidence that the cola lost sweetness in storage?*

DataDesk

```
cola:
Test Ho: mu (cola) = 0 vs Ha: mu (cola) > 0
Sample Mean = 1.02000 t-Statistic = 2.697 w/9 df
Reject Ho at Alpha = 0.0500
p = 0.0123
```

Minitab

Session

```
TEST  OF  MU  =  0.000  VS  MU  G.  T.  0.000
          N    MEAN    STDEV   SE MEA   T    P  VALUE
cola      10   1.02    1.196     0.3   2.7     0.012
```

CrunchIt!

One sample T statistics

T-test results:
μ - mean of Variable
$H_0 : \mu = 0$
$H_A : \mu > 0$

| Variable | Sample Mean | Std. Err | DF | T-Stat | P-value |
|---|---|---|---|---|---|
| cola | 1.02 | 0.37824154 | 9 | 2.6966896 | 0.0123 |

Fathom

Test of sweetness — Test Mean

Attribute (numeric): SweetLoss

Ho: population mean of **SweetLoss** equals **0**
Ha: population mean of **SweetLoss is greater than 0**

| | |
|---|---|
| Count: | **10** |
| Mean: | **1.02** |
| Std dev: | **1.1961** |
| Std error: | **0.378242** |
| Student's t: | **2.697** |
| DF: | **9** |
| P-value: | **0.012** |

**12.13 No-fee credit card offer, I** A bank wonders whether omitting the annual credit card fee for customers who charge at least $2400 in a year would increase the amount charged on its credit cards. The bank makes this offer to an SRS of 200 of its credit card customers. It then compares how much these customers charge this year with the amount that they charged last year. The mean increase is $332, and the standard deviation is $108.

(a) Is there significant evidence at the 1% level that the mean amount charged increases under the no-fee offer? Give appropriate statistical evidence to support your conclusion.

(b) Construct and interpret a 99% confidence interval for the mean amount of the increase.

**Section II** → AP STATS EXAM 2007

4. Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of *E. coli* bacteria contamination in beef. Two different methods (A and B) of determining the level of contamination were used on each of ten randomly selected specimens of a certain type of beef. The data obtained, in millimicrobes/liter of ground beef, for each of the methods are shown in the table below.

| | | Specimen | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Method | A | 22.7 | 23.6 | 24.0 | 27.1 | 27.4 | 27.8 | 34.4 | 35.2 | 40.4 | 46.8 |
| | B | 23.0 | 23.1 | 23.7 | 26.5 | 26.6 | 27.1 | 33.2 | 35.0 | 40.5 | 47.8 |

Is there a significant difference in the mean amount of *E. coli* bacteria detected by the two methods for this type of beef? Provide a statistical justification to support your answer.



GO ON TO THE NEXT PAGE.

AP STATS NOTES [12.1] #1

NAME PER

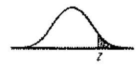
### z Test for a Population Mean

To test the hypothesis $H_0$: $\mu = \mu_0$ based on an SRS of size $n$ from a population with unknown mean $\mu$ and known standard deviation $\sigma$, compute the one-sample z statistic

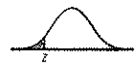
$$\boxed{z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}}$$ CH 11

In terms of a variable Z having the standard Normal distribution, the P-value for a test of $H_0$ against

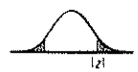
$H_a$: $\mu > \mu_0$ is $P(Z \geq z)$




$H_a$: $\mu < \mu_0$ is $P(Z \leq z)$




$H_a$: $\mu \neq \mu_0$ is $P(Z \geq |z|)$




These P-values are exact if the population distribution is Normal and are approximately correct for large $n$ in other cases.

→ POPULATION STANDARD DEVIATION
$\boxed{\sigma}$ IS KNOWN

→ POPULATION STANDARD DEVIATION
$\boxed{\sigma}$ IS UNKNOWN

→ USE $\boxed{s}$ SAMPLE STANDARD DEVIATION

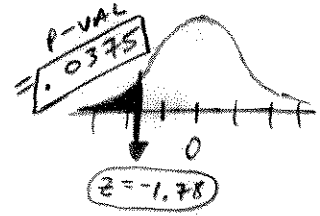
### The One-Sample t Statistic and the t Distributions

Draw an SRS of size $n$ from a population that has the Normal distribution with mean $\mu$ and standard deviation $\sigma$. The one-sample $t$ statistic

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$ CH 12

has the $t$ distribution with $n - 1$ degrees of freedom.

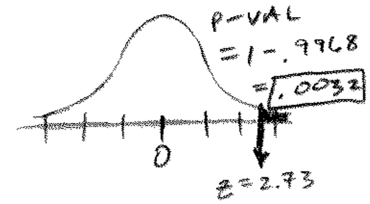
1. The one-sample z statistic for a one-sided test has the value z = - 1.78
Find the P-value for the test.
Is this significant at the 10% level?
Is this significant at the 5% level?




YES, SIGNIF at 10% LEVEL
B/C P-VAL .0375 ≤ .10

YES, SIGNIF at 5% LEVEL
B/C P-VAL .0375 ≤ .05

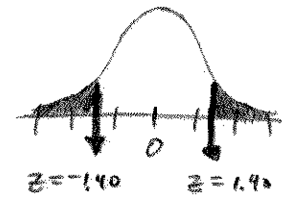
2. The one-sample z statistic for a one-sided test has the value z = 2.73
Find the P-value for the test.
Is this significant at the 5% level?
Is this significant at the 1% level?




YES, SIGNIF at 5% LEVEL
B/C P-VAL .0032 ≤ .05

YES, SIGNIF at 1% LEVEL
B/C P-VAL .0032 ≤ .01

3. The one-sample z statistic for a two-sided test has the value z = 1.40
Find the P-value for the test.
Is this significant at the 10% level?
Is this significant at the 5% level?




P-VAL = SHADED AREA
= 2 * (.0808)
= .1616

NOT SIGNIF at 10% LEVEL
B/C P-VAL .1616 NOT ≤ .10

NOT SIGNIF at 5% LEVEL
B/C P-VAL .1616 NOT ≤ .05

**Using the "t table," I** What critical value $t^*$ from Table C satisfies each of the following conditions?

(a) The $t$ distribution with 9 degrees of freedom has probability .02 to the right of $t^*$. → $t^* = 2.398$

(b) The $t$ distribution with 16 degrees of freedom has probability .95 to the left of $t^*$. → $t^* = 1.746$

**Using the "t table," II** What critical value $t^*$ from Table C satisfies each of the following conditions?

(a) The one-sample $t$ statistic from a sample of 25 observations has probability .005 to the right of $t^*$.

df = 24
$t^* = 2.797$

(b) The one-sample $t$ statistic from an SRS of 14 observations has probability .8 to the left of $t^*$.

df = 13
$t^* = .870$

## The One-Sample t Test    σ unknown ✓

Draw an SRS of size $n$ from a population having unknown mean $\mu$. To test the hypothesis $\boxed{H_0: \mu = \mu_0}$ based on an SRS of size $n$, compute the one-sample $t$ statistic

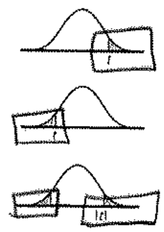
$$\boxed{t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}} \qquad df = n - 1$$

In terms of a random variable $T$ having the $t(n-1)$ distribution, the $P$-value for a test of $H_0$ against

$H_a: \mu \boxed{>} \mu_0$ is $P(T \geq t)$

$H_a: \mu \boxed{<} \mu_0$ is $P(T \leq t)$

$H_a: \mu \boxed{\neq} \mu_0$ is $2P(T \geq |t|)$




These $P$-values are exact if the population distribution is Normal and are approximately correct for large $n$ in other cases.

- CONDITIONS FOR ONE-SAMPLE t TEST
  (1) SRS, (2) NORMALITY, (3) INDEPENDENCE  N ≥ 10n

IF POPULATION DISTRIBUTION IS NOT NORMAL THEN ... IF n < 15, DATA MUST BE CLOSE TO NORMAL
IF n ≥ 15, DATA CAN BE SOMEWHAT SKEWED BUT NO OUTLIERS
IF n ≥ 30 DATA CAN BE STRONGLY SKEWED

**One-sided t test** The one-sample $t$ statistic for testing

$H_0: \mu = 0$

$H_a: \mu > 0$

from a sample of $n = 20$ observations has the value $t = 2.46$


(a) What are the degrees of freedom for this statistic? $\boxed{df = 19}$

(b) Give the two critical values $t^*$ from Table ~~C~~ B that bracket $t$. What are the upper tail probabilities $p$ for these two entries? $\boxed{t = 2.205 \text{ \& } t = 2.539}$

(c) Between what two values does the $P$-value of the test fall? $\boxed{.02 \text{ \& } .01}$

(d) Is the value $t = 2.46$ significant at the 5% level? Is it significant at the 1% level?

YES, SIGNIF at 5% LEVEL b/c BETWEEN .01 & .02 ≤ .05 — REJECT $H_0$ at 5% LEVEL

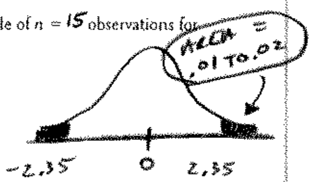
NO, NOT SIGNIF at 1% LEVEL b/c BETWEEN .01 & .02 NOT ≤ .01 — FAIL TO REJECT $H_0$ at 1% LEVEL

**Two-sided t test** The one-sample $t$ statistic from a sample of $n = 15$ observations for the two-sided test of

$H_0: \mu = 64$

$H_a: \mu \neq 64$

has the value $t = 2.35$




(a) What are the degrees of freedom for $t$? $\boxed{df = 14}$

(b) Locate the two critical values $t^*$ from Table ~~C~~ B that bracket $t$. What are the upper tail probabilities $p$ for these two values? $\boxed{t = 2.264 \text{ \& } t = 2.624}$

(c) Between what two values does the $P$-value of the test fall? (Note that $H_a$ is two-sided.)

(d) Is the value $t = 2.35$ statistically significant at the 10% level? At the 5% level?

P-VALUE = BETWEEN .02 & .04

SIGNIF at 10% LEVEL b/c .02 TO .04 ≤ .10
THUS REJECT $H_0$ at the 10% SIGNIF LEVEL

SIGNIF at 5% LEVEL b/c .02 TO .04 ≤ .05
THUS REJECT $H_0$ at the 5% SIGNIF LEVEL

**Using Table ~~C~~ B** The one-sample $t$ statistic for a test of

$H_0: \mu = 10$

$H_a: \mu < 10$

based on $n = 30$ observations has the value $t = -2.37$


(a) What are the degrees of freedom for this statistic? $\boxed{df = 29}$

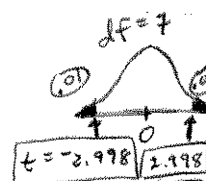
(b) Between what two probabilities from Table ~~C~~ B does the $P$-value of the test fall?

LOOK FOR t = 2.37 ?

t = 2.150 & t = 2.462

P-VALUE = BETWEEN .01 & .02

**Significance** You are testing $H_0: \mu = 0$ against $\boxed{H_a: \mu \neq 0}$ based on an SRS of 8 observations from a Normal population. What values of the $t$ statistic are statistically significant at the $\alpha = .02$ level? df = 7



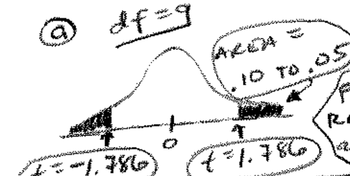

$t = \pm 2.998$

**Sample size and significance** For a sample of size 10 a test of a null hypothesis versus a two-sided alternative gives $t = 1.786$  df = 9

(a) Is the test result significant at the 5% level? Draw a sketch of the appropriate $t$ distribution and illustrate your calculation with this sketch.

(b) Now assume that the same test statistic was obtained for a sample size of $n = 20$. Assess the statistical significance of the result and illustrate the calculation with a sketch. How did the statistical significance change with the sample size? Explain your answer.  df = 19

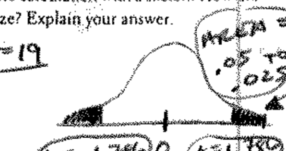
(a) df = 9




P-VALUE = .10 TO .20

FAIL TO REJECT $H_0$ at 5% LEVEL

(b) df = 19




P-VALUE = .05 TO .10
SIGNIF at 10% LEVEL

AP STATS NOTES [12.1] #2 NAME PER

**Example 12.2** *Sweet cola*
Performing a one-sample $t$ test


1-VAR STATS

$\bar{x} = 1.02$
$s_x = 1.1961$
$n = 10$

Diet colas use artificial sweeteners to avoid sugar. These sweeteners gradually lose their sweetness over time. Manufacturers therefore test new colas for loss of sweetness before marketing them. Trained tasters sip the cola along with drinks of standard sweetness and score the cola on a "sweetness scale" of 1 to 10. The cola is then stored for a month at high temperature to imitate the effect of four months' storage at room temperature. Each taster scores the cola again after storage. Our data are the differences (score before storage minus score after storage) in the tasters' scores. The bigger these differences, the bigger the loss of sweetness. Here are the sweetness losses for a new cola, as measured by 10 trained tasters:

• PUT #'S INTO $\boxed{L1}$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2.0 | 0.4 | 0.7 | 2.0 | −0.4 | 2.2 | −1.3 | 1.2 | 1.1 | 2.3 |

Most are positive. That is, most tasters found a loss of sweetness. But the losses are small, and two tasters (the negative scores) thought the cola gained sweetness. *Are these data good evidence that the cola lost sweetness in storage?*

$\mu_{DIFF} \rightarrow$ TRUE MEAN SWEETNESS LOSS

I HYPOTHESES $\mu_{DIFF} = \mu_{BEFORE} - \mu_{AFTER}$

$H_0: \mu_{DIFF} = 0 \rightarrow$ NO SWEETNESS LOSS

$H_a: \mu_{DIFF} > 0 \rightarrow$ THERE IS A LOSS

II CONDITIONS $\rightarrow$ ONE-SAMPLE $t$-TEST B/C $\sigma$ UNKNOWN

① SRS $\rightarrow$ NOT STATED, BUT TRAINED TASTERS WERE USED, TREAT AS AN SRS ✓

② NORMALITY $\rightarrow$ SMALL SAMPLE SIZE ($n = 10 < 15$) BUT THE SAMPLE DATA GRAPH IS CLOSE TO NORMAL ✓

③ INDEPENDENCE

$\rightarrow$ ASSUME 10 TASTE TESTERS DO NOT AFFECT EACHOTHER

III CALCULATIONS

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.02 - 0}{1.1961/\sqrt{10}} = \boxed{2.6967}$$

$df = 9$


P-VALUE = $\boxed{\text{BETWEEN } .02 \ \& \ .01}$

$\boxed{\text{REJECT } H_0 \text{ B/C } .01 \text{ TO } .02 \leq .05}$

WE CONCLUDE THE COLA LOST SWEETNESS IN STORAGE

CALCULATOR $\rightarrow t = 2.6967$, P-VALUE $= .0123$

DataDesk

```
cola:
Test Ho: mu (cola) = 0 vs Ha: mu (cola) > 0
Sample Mean = 1.02000 t-Statistic = 2.697 w/9 df
Reject Ho at Alpha = 0.0500
p = 0.0123
```

Minitab

Session

$H_0: \mu = 0$ $H_a: \mu > 0$

```
TEST  OF  MU = 0.000  VS  MU  G.  T.  0.000
          N    MEAN   STDEV   SE MEA    T    P VALUE
cola     10    1.02   1.196     0.3   2.7     0.012
```

VAR $n$ $\bar{x}$ $s$ $s/\sqrt{n}$

CrunchIt!

One sample T statistics

T-test results:
μ : mean of Variable
$H_0: \mu = 0$
$H_A: \mu > 0$

| Variable | Sample Mean | Std. Err | DF | T-Stat | P-value |
|---|---|---|---|---|---|
| cola | 1.02 | 0.37824154 | 9 | 2.6966896 | 0.0123 |

Fathom

Test of sweetness — Test Mean

Attribute (numeric): SweetLoss

Ho: population mean of **SweetLoss** equals 0
Ha: population mean of **SweetLoss** is greater than 0

$H_0: \mu = 0$
$H_a: \mu > 0$

| | | |
|---|---|---|
| Count: | 10 | $\rightarrow n$ |
| Mean: | 1.02 | $\rightarrow \bar{x}$ |
| Std dev: | 1.1961 | $\rightarrow s$ |
| Std error: | 0.378242 | $\rightarrow s/\sqrt{n}$ |
| Student's t: | 2.697 | $\rightarrow t$ |
| DF: | 9 | |
| P-value: | $\boxed{0.012}$ | |

**12.13 No-fee credit card offer, I** A bank wonders whether omitting the annual credit card fee for customers who charge at least \$2400 in a year would increase the amount charged on its credit cards. The bank makes this offer to an SRS of 200 of its credit card customers. It then compares how much these customers charge this year with the amount that they charged last year. The mean increase is \$332, and the standard deviation is \$108.

(a) Is there significant evidence at the 1% level that the mean amount charged increases under the no-fee offer? Give appropriate statistical evidence to support your conclusion.

(b) Construct and interpret a 99% confidence interval for the mean amount of the increase.

(a) $n = 200$, $\bar{x} = 332$, $s = 108$

I HYPOTH

$H_0: \mu_d = 0$

$H_a: \mu_d > 0$

$\mu_d \rightarrow$ TRUE MEAN DIFFERENCE IN AMOUNT CHARGED ON CREDIT CARDS AFTER OMITTING ANNUAL FEE

II CONDITIONS ONE-SAMPLE $t$ TEST B/C $\sigma$ UNKNOWN

① SRS → of 200 ✓ ② NORMALITY ✓ $n = 200 \geq 30$ ✓

③ INDEPENDENCE $N \geq 10n$ # OF CUSTOMERS $\geq 2000$ ✓

III CALCULATIONS $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{332 - 0}{108/\sqrt{200}} = \boxed{43.47}$

$df = 199$

USE $df = 100$

$\boxed{P\text{-VALUE} \approx 0}$

WAY OFF CHART

0

43.47

IV INTERPRETATION

WE REJECT $H_0$ B/C P-VALUE $\approx 0 \leq .01$

WE CAN CONCLUDE WITH STRONG EVIDENCE THAT THE MEAN AMOUNT CHARGED WOULD INCREASE WITH THE "NO FEE" OFFER

(b) 99% CONF INTERVAL, USE $df = 100$ FROM TABLE

$CI = \bar{x} \pm t^* \frac{s}{\sqrt{n}}$

$= 332 \pm (2.626) \frac{108}{\sqrt{200}}$

$= 332 \pm 20.05 = \boxed{(311.95, 352.05)}$

→ IS THIS CHANGE CAUSED BY THE "ECONOMY" OR BY THE "NO ANNUAL FEE" OFFER?

**Section II** → AP STATS EXAM 2007

4. Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of *E. coli* bacteria contamination in beef. Two different methods (A and B) of determining the level of contamination were used on each of ten randomly selected specimens of a certain type of beef. The data obtained, in millimicrobes/liter of ground beef, for each of the methods are shown in the table below.

$n = 10$

| | Specimen | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Method | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A | 22.7 | 23.6 | 24.0 | 27.1 | 27.4 | 27.8 | 34.4 | 35.2 | 40.4 | 46.8 |
| B | 23.0 | 23.1 | 23.7 | 26.5 | 26.6 | 27.1 | 33.2 | 35.0 | 40.5 | 47.8 |
| A−B → | −.3 | .5 | .3 | .6 | .8 | .7 | 1.2 | .2 | −.1 | −1.0 |

Is there a significant difference in the mean amount of *E. coli* bacteria detected by the two methods for this type of beef? Provide a statistical justification to support your answer.

✗ TOP IS USUALLY BIGGER, USE METHOD A − METHOD B

✗ USE A MATCHED PAIRS DESIGN TO PERFORM A HYPOTHESIS TEST FOR THE MEAN DIFFERENCE IN LEVEL OF E. COLI BACTERIA CONTAMINATION IN BEEF DETECTED BY THE TWO METHODS A & B

I $H_0: \mu_d = 0$

$H_a: \mu_d \neq 0$ ✓

$\mu_d$ IS THE MEAN DIFFERENCE (METHOD A − METHOD B) IN THE E. COLI LEVEL OF BACTERIA CONTAMINATION IN BEEF DETECTED BY THE TWO METHODS ✓

II CONDITIONS

① SRS ✓ SPECIMENS ARE RANDOMLY SELECTED

② NORMALITY ✓ $n = 10 < 15$ IS SMALL, BUT SAMPLE DATA ARE APPROXIMATELY NORMAL, OK TO USE $t$-TEST

③ INDEPENDENCE ✓

B/C SPECIMENS ARE RANDOMLY SELECTED, IT IS REASONABLE TO ASSUME 10 DATA PAIRS ARE INDEPENDENT OF ONE ANOTHER

III CALCULATIONS PAIRED $t$-TEST $\boxed{t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}}$

$\bar{x} = .29$, $s = .629727$, $df = 9$

$t = \frac{.29 - 0}{.629727/\sqrt{10}} = \boxed{1.46}$ ✓ P-VALUE $= 2 * (.10 \text{ \& } .05)$ BETWEEN $\boxed{.10 \text{ TO } .20}$ ✓ $\boxed{\approx .1793}$ ✓

64

GO ON TO THE NEXT PAGE.

IV B/C P-VALUE $\approx .1793$ IS NOT $\leq .05$, WE FAIL TO REJECT $H_0$ SO WE DO NOT HAVE STATISTICALLY SIGNIFICANT EVIDENCE TO CONCLUDE THERE IS A DIFFERENCE IN MEAN AMOUNT OF E. COLI DETECTED