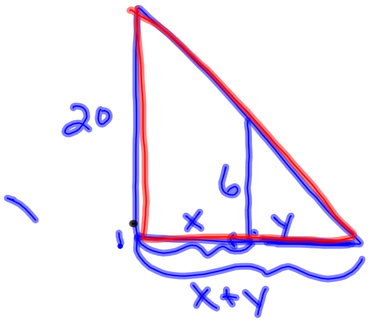
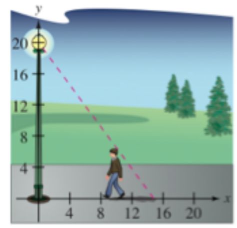
Example 5: A man 6 feet tall walks a rate of 5 feet per second toward a light that is 20 feet above the ground. At what rate is the tip of his shadow changing?




Given: $\frac{dx}{dt} = -5 \frac{ft}{sec}$

want: $\frac{dy}{dt}$



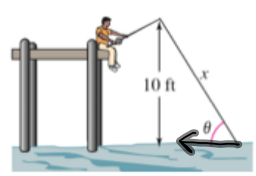
Use similar Δ's

$$\frac{6}{y} = \frac{20}{x+y}$$

$$6x + 6y = 20y$$
$$6x = 14y$$
$$6\frac{dx}{dt} = 14\frac{dy}{dt}$$
$$\frac{3}{7}\frac{dx}{dt} = \frac{dy}{dt}$$
$$\frac{dy}{dt} = \frac{3}{7}(-5) = \frac{-15}{7}\frac{ft}{s}$$

$$\frac{d(x+y)}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$$
$$-5 - 15/7 = \frac{-50}{7} \frac{ft}{sec}$$

Example 6: A fish is reeled in at a rate of 1 foot per second from a point 10 feet above the water. At what rate is the angle of elevation between the line and the water changing when there is a total of 25 feet of line out?



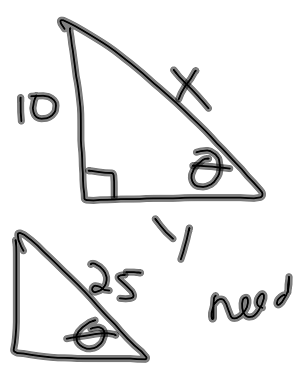

Given: $\frac{dx}{dt} = -1 \frac{ft}{sec}$ want: $\frac{d\theta}{dt}$

when $x = 25 ft$

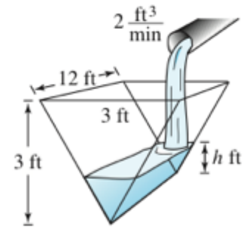
$$\sin\theta = \frac{10}{x}$$

$$\cos(\theta) \cdot \frac{d\theta}{dt} = -10x^{-2}\frac{dx}{dt}$$

$$\cos(\theta)\boxed{\frac{d\theta}{dt}} = -\frac{10}{x^2} \cdot \frac{dx}{dt}$$



Example 7: A trough is 12 feet long and 3 feet across the top. Its ends are isosceles triangles with altitudes of 3 feet. If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when h is 1 foot deep ?




**37.** ***Machine Design*** The endpoints of a movable rod of length 1 meter have coordinates $(x, 0)$ and $(0, y)$ (see figure). The position of the end on the $x$-axis is

$$x(t) = \frac{1}{2}\sin\frac{\pi t}{6}$$

where $t$ is the time in seconds.

(a) Find the time of one complete cycle of the rod.

(b) What is the lowest point reached by the end of the rod on the $y$-axis?

(c) Find the speed of the $y$-axis endpoint when the $x$-axis endpoint is $\left(\frac{1}{4}, 0\right)$.